

Coupling Right- and Left-Handed Photons Differently to Charged Matter

Brett Altschul¹

*Department of Physics and Astronomy
University of South Carolina
Columbia, SC 29208 USA*

Abstract

We consider a modification of electrodynamics in which right- and left-circularly polarized photons are coupled to charged sources differently. Even though photon helicity is a Lorentz invariant quantity, such a modification breaks Lorentz symmetry, as well as locality. The modified theory includes novel magnetic forces between perpendicular currents. Existing data can be used to constrain the modification at approximately a 2×10^{-3} level.

¹baltschu@physics.sc.edu

1 Introduction

In recent years, there has been a great surge in interest in the possibility of Lorentz violation [1]. The exploration of possible deviations from special relativity has both important experimental and theoretical facets. There are many different forms that Lorentz violation could take and several different approaches to the questions surrounding them.

There are a number of possible motivations for focusing on a particular modification of known physics. The minimal SME approach [2] considers a theory containing all possible local, gauge-invariant, superficially renormalizable operators involving only standard model fields—even when the operators are not Lorentz invariant. The number of such operators, unrestricted by Lorentz symmetry, is quite large. Alternatively, one might consider, in a unsystematic way, only certain operators that have particularly interesting or attractive structures.

In this paper, we shall look at a form of Lorentz violation that has been selected for yet a different reason. Rather than beginning with a particular form of interaction, we start with a physical phenomenon and ask what kind of interaction could produce it. There are immediately two lines of inquiry that can be followed: examining what impact the existence of the novel phenomenon must have in other regimes and determining what are the experimental limits on the phenomenon. The first line of inquiry entails developing a predictive quantum theory that contains the novel phenomenon. In general, the choice of such a theory will not be unique. However, there may exist a particular version which is clearly the simplest or has the most desirable properties, whereas other formulations may turn out to have pathologies.

In 1990, Carroll, Field, and Jackiw introduced a Lorentz-violating modification of electromagnetism with the gauge field Lagrange density

$$\mathcal{L}_{CS} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}k_\mu\epsilon^{\mu\nu\rho\sigma}F_{\nu\rho}A_\sigma. \quad (1)$$

k is an externally prescribed four-vector. This modification, which has a Chern-Simons form [3, 4, 5], causes right- and left-circularly polarized electromagnetic waves to propagate at different phase speeds. The birefringence that would result from such an effect has not been seen, even for light coming from cosmological distances [3, 6, 7]. Yet while this kind of Chern-Simons term is not experimentally viable, it has stirred a great deal of interest in similar theories.

Searching for an analogous modification of general relativity, Jackiw and Pi [8] looked at a model (which was also considered in [9]) with a gravitational Lagrange density

$$\mathcal{L}_G = \frac{1}{16\pi G} \left[\sqrt{-g}R - \frac{1}{2}v_\mu\epsilon^{\mu\nu\rho\sigma} \left(\Gamma_{\nu\beta}^\alpha\partial_\rho\Gamma_{\sigma\alpha}^\beta + \frac{2}{3}\Gamma_{\nu\beta}^\alpha\Gamma_{\rho\gamma}^\beta\Gamma_{\sigma\alpha}^\gamma \right) \right], \quad (2)$$

which contains the analogous gravitational Chern-Simons term. (The divergence of the quantity contracted with v is proportional to the Riemann tensor contracted with its dual.)

However, in this case, if diffeomorphism invariance is enforced, the apparent Lorentz violation coming from the prescribed v is illusory [10]; Lorentz symmetry cannot be broken in a diffeomorphism invariant theory [11]. Nonetheless, the added term does have physical consequences. The two polarizations of gravitational waves must travel at the same speed because of boost invariance, but the circularly polarized radiation states couple differently to the energy-momentum tensor. In accordance with the appearance of the Levi-Civita tensor ϵ in the modified action, this modification violates parity.

So, inspired by the electromagnetic Chern-Simons theory (in which the two polarizations propagate differently) a similar-looking gravitational theory (in which the polarizations couple differently) was introduced. The goal of this paper is to examine a somewhat analogous theory to the one described by \mathcal{L}_G , but back in the realm of quantum electrodynamics (QED). That is, we shall look at a modification of QED that causes the two photon helicities to couple differently to charged matter. Taking the simplest such model, we shall find that our model, like the Chern-Simons theory \mathcal{L}_{CS} , violates Lorentz boost symmetry.

This paper is organized as follows. In section 2, the basic theory with different couplings for electromagnetic waves of different helicities is discussed. Section 3 shows how the novel interaction may be recast as a change to the photon propagator. Then in section 4, the modified propagator is used to derive an expression for a new potential existing between perpendicular current elements. Section 5 discusses the question of experimental constraints on the modified theory and presents the paper's final conclusions.

2 QED with Helicity-Dependent Couplings

In the Chern-Simons gravity theory described by \mathcal{L}_G , the coupling of the gravitons to their sources depends not only on helicity but also on frequency. This introduces a new dimensional scale μ . Since gravitation is already described by a nonrenormalizable theory, the introduction of such a scale, accompanied in the action by additional derivatives, does not necessarily worsen the behavior of the theory. However, QED is a renormalizable theory, and adding extra derivatives to the coupling term would presumably destroy this feature; the introduction of an operator with mass dimension greater than four would generate pathologies. For this reason, we shall consider a modification of electromagnetism in which the photon coupling depends on helicity but not on frequency and in which there is no new dimensional scale. (A similar modification of gravity was considered in [12]. The theory had the differing right- and left-handed couplings of the Chern-Simons theory, but without a new dimensional scale. Such a modification to gravity could lead to changes to the cosmic microwave background polarization.)

The division of photons into right- and left-circularly polarized is superficially Lorentz invariant. No rotation or boost will change the helicity of a given photon mode. Nonetheless, it does not appear to be possible to assign different couplings to the right- and

left-handed photons in Lorentz-invariant fashion. The reason is that the photon portion of the electromagnetic field cannot be disentangled from the electrostatic portion, and there is no analogous Lorentz-invariant separation of the electrostatic interaction. It is possible to separate the full electromagnetic sector into right- and left-handed parts in the absence of charges, when the electrostatic part of the Hamiltonian vanishes. However, what remains is a free theory, for which the coupling is not an observable parameter.

Our starting point will be the electromagnetic Hamiltonian in the Coulomb gauge, $\vec{\nabla} \cdot \vec{A} = 0$, which is

$$\begin{aligned} H &= H_0 + H_1 + H_2 \\ &= \frac{1}{2} \int d^3x \left(|\vec{\nabla} \times \vec{A}|^2 + |\partial \vec{A} / \partial t|^2 \right) - \int d^3x \vec{j} \cdot \vec{A} + \frac{1}{2} \int d^3x \int d^3x' \frac{\rho(\vec{x}, t) \rho(\vec{x}', t)}{4\pi |\vec{x} - \vec{x}'|}. \end{aligned} \quad (3)$$

When the charge density is carried by elementary charged particles, H_1 is $\mathcal{O}(e)$ while H_2 is already $\mathcal{O}(e^2)$. We use this particular form of H because it separates the electrostatic and propagating parts of the electromagnetic field. Such a separation is needed, because the electrostatic field does not have a decomposition into right- and left-handed parts the way the radiation field does.

In fact, in this gauge the electrostatic potential is a constrained degree of freedom. A_0 has been eliminated from the Hamiltonian in favor of the double integral term H_2 . (Note that for point charges, this Coulomb repulsion term includes the infinite self-energies of individual particles.) The remaining \vec{A} is constrained by the Coulomb condition, leaving two physical degrees of freedom. These two degrees of freedom contain the photon modes as well as all magnetostatic effects. The residual gauge symmetry of this Hamiltonian is that we may add to \vec{A} the gradient of a harmonic function.

Our modification shall be splitting the photon field into two separate parts, with different helicities, and coupling them differently to the current \vec{j} . Taking

$$\vec{A}(\vec{x}, t) = \vec{A}_+(\vec{x}, t) + \vec{A}_-(\vec{x}, t), \quad (4)$$

the two halves of \vec{A} are

$$\vec{A}_\pm(\vec{x}, t) = \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_{\vec{q}}} \left[a_{\vec{q}, \pm} \vec{\epsilon}^{(\pm)} e^{-iq \cdot x} + a_{\vec{q}, \pm}^\dagger \vec{\epsilon}^{(\pm)*} e^{iq \cdot x} \right]. \quad (5)$$

The circular polarization vectors are $\vec{\epsilon}^{(\pm)}(\vec{q}) = \mp \frac{1}{\sqrt{2}} [\vec{\epsilon}^{(1)} \pm i\vec{\epsilon}^{(2)}]$, where $[\vec{\epsilon}^{(1)}, \vec{\epsilon}^{(2)}, \hat{q}]$ form a right-handed triplet; and q_0 in the four-vector dot product $q \cdot x$ is $q_0 = \omega_{\vec{q}} = |\vec{q}|$.

We then replace the conventional H_1 with

$$H'_1 = - \sum_{\pm} \frac{(1 \pm \kappa)}{\sqrt{1 + \kappa^2}} \int d^3x \vec{j} \cdot \vec{A}_\pm. \quad (6)$$

Thus right- and left-handed photons couple to the current with different strengths; the difference is determined by the (small) parameter κ . The insertion of $1/\sqrt{1 + \kappa^2}$ might

seem peculiar, but it is related to the fact that there are actually three coupling constants present in the theory now—the couplings for right- and left-handed photons, $e_{(+)}$ and $e_{(-)}$, and also the electrostatic coupling $e_{(0)}$ that appears in H_2 . It seems obvious that $e_{(0)}$ should be some kind of average of $e_{(+)}$ and $e_{(-)}$, and as we shall see, the most natural relationship among the three is

$$e_{(0)}^2 = \frac{1}{2}[e_{(+)}^2 + e_{(-)}^2]. \quad (7)$$

This relationship is embodied in the modified coupling (6).

In studies of Lorentz violation, it is important to distinguish between two types of Lorentz transformations. “Observer” (or “passive”) transformations correspond merely to relabeling of coordinates, and all theories should be invariant under this kind of reparameterization. “Particle” (or “active”) transformations, on the other hand, are physically meaningful. While an observer rotation merely means studying an experiment in rotated coordinates, the particle rotation corresponds to actually rotating the experimental apparatus into a different orientation. The simplest phenomena in Lorentz-violating physics are preferred frame effects. The preferred frame is one in which the physics are invariant under rotations but not under boosts. The form of Lorentz violation discussed here is clearly of the preferred frame type. More generally, if a type of Lorentz violation may be completely parameterized by a single timelike four-vector w^μ , there is always an isotropic preferred frame—the one in which the spatial components of w vanish.

Moreover, the discrete symmetries of the modification are easy to determine. The conventional electromagnetic coupling is invariant under parity (P), charge conjugation (C), and time reversal (T). The new term added to the Hamiltonian has the same form, except it includes an extra factor of the photon helicity. The helicity is odd under P and even under C and T; the novel interaction inherits these same discrete symmetries and is hence odd under CPT. This is actually not surprising, since we shall see that this form of Lorentz violation can indeed be described by a single preferred timelike vector. The usual expectation is that forms of Lorentz violation described by background tensors with odd numbers of Lorentz indices should be odd under CPT, while those with even numbers of indices should be even under CPT. While this correspondence does not necessarily hold outside the scope of local Lagrangian field theory [13], it does hold in this case.

3 Electromagnetic Propagator

In perturbation theory, the fundamental objects of interest are the propagators and the vertex factors. The form of Lorentz violation considered here—although it appears as a modification of the interaction—can be absorbed into a modified photon propagator. In the noncovariant approach based on the Hamiltonian H , the photon propagator must be assembled from matrix elements of both H_1 and H_2 . At $\mathcal{O}(e^2)$, a matrix element receives a second-order contribution from two factors of H_1 and a first-order contribution from

H_2 . These fit together to yield a covariant result. (The procedure is outlined in [14], for example.) The H_1 part of the calculation involves evaluating

$$\tilde{D}_{jk}(x-x') \equiv \langle 0|T\{A_j(x)A_k(x')\}|0\rangle = \int \frac{d^4q}{(2\pi)^4} \delta_{jk}^T \frac{i}{q^2 + i\epsilon} e^{-iq \cdot (x-x')}. \quad (8)$$

δ_{jk}^T is the transverse δ -function, $\delta_{jk}^T = \delta_{jk} - \hat{q}_j \hat{q}_k$.

With the parity-violating $H' = H_0 + H'_1 + H_2$, the only change to the propagator calculation is that we must evaluate

$$\tilde{D}'_{jk}(x-x') = \langle 0|T\left\{ \frac{(1+\kappa)^2}{1+\kappa^2} [A_+(x)]_j [A_+(x')]_k + \frac{(1-\kappa)^2}{1+\kappa^2} [A_-(x)]_j [A_-(x')]_k \right\} |0\rangle. \quad (9)$$

(The cross terms with $A_+ A_-$ vanish.) In including the κ -dependent factors as part of \tilde{D}'_{jk} , we are effectively absorbing these factors into the definition of the electromagnetic field. This can also be accomplished directly with a nonlocal field redefinition, which moves the new physics from the interaction term H_1 to the photon kinetic term H_0 .

In passing from \tilde{D}_{jk} to \tilde{D}'_{jk} , the only modifications needed are to the expression's Lorentz structure. For \tilde{D}'_{jk} , we must evaluate

$$\sum_{\pm} \frac{(1 \pm \kappa)^2}{1 + \kappa^2} \epsilon_j^{(\pm)} \epsilon_k^{(\pm)*} = \delta_{jk}^T - 2i \frac{\kappa}{1 + \kappa^2} \epsilon_{jkl} \hat{q}_l. \quad (10)$$

The δ_{jk}^T term combines with the electrostatic contribution to give the usual photon propagator; the $1/\sqrt{1+\kappa^2}$ in (6) ensures the normalization of this contribution is correct. The other term in (10) generates a new contribution to the effective propagator. In momentum space, this propagator is

$$D'_{\mu\nu}(q) = -i \frac{g_{\mu\nu} - (1-\zeta) q_\mu q_\nu / q^2 - 2i \frac{\kappa}{1+\kappa^2} \hat{q}^\beta \epsilon_{0\beta\mu\nu}}{q^2 + i\epsilon}. \quad (11)$$

The four-vector \hat{q}^μ can be either $(0, \hat{q})$ or $(q_0/|\vec{q}|, \hat{q}) = q^\mu/|\vec{q}|$. The gauge parameter ζ appears here even though we derived the propagator from a gauge-fixed Lagrangian, because the term it multiplies has no effect when contracted with conserved currents. If this were to be a viable modification of QED, the effective propagator (11) ought to describe essentially all the new physics that arise in the model. The different couplings at the vertices for the two helicities have been converted into differing field strengths during propagation.

While (11) provides a complete description of the tree-level photon propagator, it is also natural to ask how the electromagnetic two-point function is affected by quantum corrections. At one-loop order (and leading order in κ), the photon self-energy diagram contains no internal photon propagators, and so the vacuum polarization tensor $\Pi^{\mu\nu}(q) =$

$(q^2 g^{\mu\nu} - q^\mu q^\nu) \Pi(q^2)$ is unchanged from the standard theory. In the Landau gauge ($\zeta = 1$), the insertion of a virtual fermion-antifermion loop into the photon propagator merely multiplies the propagation amplitude (11) by the q^2 -dependent renormalization factor $\Pi(q^2)$. As a consequence, one-loop quantum corrections do not introduce any new Lorentz structures in the two-point correlation function, nor do they affect the relative magnitudes of the Maxwell term and the novel κ -dependent term.

We note, moreover, that the new term in the numerator of (11) does not violate gauge invariance in any obvious way. It is transverse, since $q^\mu (\hat{q}^\beta \epsilon_{0\beta\mu\nu}) = 0$. The new interactions were introduced into a Hamiltonian that was already gauge fixed, so it is not possible to answer unambiguously whether they are gauge invariant. However, the theory appear to have two physical photon polarizations for each \vec{q} , a transverse propagator, and no new dimensional constants—all characteristic of a renormalizable theory.

However, the theory definitely does not possess Lorentz invariance; this is evident from the appearance of the specific index 0 in $\epsilon_{0\beta\mu\nu}$. The propagator could be rewritten using an externally prescribed four-vector $\kappa^\mu = (\kappa, \vec{0})$, but the correct generalization for a κ^μ that is spacelike is not obvious, because of the presence of the unit three-vector \hat{q} . However, for any timelike κ^μ , it is possible to perform an observer boost to eliminate the spatial components of κ^μ , leaving a theory with photon propagator $D'_{\mu\nu}$ in the boosted frame.

At leading order, this modification to the propagator is similar to a theory with a term proportional to $\epsilon_{0\beta\mu\nu} (\partial^\mu A^\delta) [(\partial^\beta / |\vec{\nabla}|) \partial^\nu A_\delta]$ (which is weakly nonlocal) added to the Lagrange density. There are differences apparent at higher orders, however. This raises the question of whether the theory discussed here can be considered local, and the answer is slightly ambiguous. The Hamiltonian H is nonlocal in its electrostatic part, but this is not the portion of the Hamiltonian that has been modified. Indeed, the contributions made by H_2 in perturbation theory contribute to the photon propagator in exactly the same way they do in conventional electrodynamics (which is certainly a local theory). The changes made to H'_1 can be described in a completely local formalism; for although separating the vector potential \vec{A} into its right- and left-circularly polarized parts requires an expansion in Fourier modes, there is no reason we cannot treat \vec{A}_+ and \vec{A}_- as the fundamental fields, which interact with \vec{j} in a completely local fashion. However, if these are the fundamental fields, the electrostatic H_2 may no longer be viewed as a manifestation of the same electromagnetic field as appears in the other terms in the Hamiltonian. It would have to represent a new interaction, fundamentally nonlocal in nature. Thus it does not appear possible to escape the nonlocality completely.

It seems that locality and Lorentz symmetry are violated in similar ways. The gauge-fixed Hamiltonian H has neither property, although it represents a theory that is ultimately both local and Lorentz invariant. Superficially, $H_1 - H'_1$, is both local and Lorentz invariant, but introducing it interferes with the subtle interplay among H_0 , H_1 , and H_2 that ensures a local, Lorentz-invariant S -matrix.

In fact, even Lorentz-violating field theories that are completely local can have prob-

lems with stability, causality, or both [15]. This has been worked out quite explicitly for the Chern-Simons theory described by \mathcal{L}_{CS} [3] with a timelike k . There are runaway solutions to the equations of motion, because the Hamiltonian is not bounded from below. One can arrange the boundary conditions such that the runaway modes are never excited. However, the Green's functions with these boundary conditions are acausal; charges begin to radiate before they actually accelerate.

On the other hand, there are nonlocal Lorentz-violating field theories that are better behaved with respect to stability and causality than the local theories [13]. It is difficult to see how the present theory could have problems with causality, since the photon modes propagate only on the light cone. Stability of the theory as a whole (including charged fermions) is not so clear, but there are certainly no runaway modes in the pure electromagnetic sector.

4 Anomalous Potential

At nonrelativistic energies, the dominant effect of the electromagnetic field is the Coulomb interaction between charges; there is also a magnetostatic potential between idealized infinitesimal current elements. Changes to the structure of the electromagnetic sector will generally produce corresponding changes in the nonrelativistic potentials. However, in the modified theory discussed here, there are not expected to be any changes to the scalar potential A_0 ; the Coulomb part of the Hamiltonian H_2 was explicit in the original formulation of the theory, and it was not modified. In contrast, there is a change to the potential between currents, which we can evaluate.

We consider two infinitesimal current elements, $d\vec{I}_1 = I_1 d\vec{l}_1$ and $d\vec{I}_2 = I_2 d\vec{l}_2$, separated by a vector \vec{r}_{12} . The nonrelativistic potential between them is the three-dimensional Fourier transform of the contraction of these currents with the effective propagator,

$$V(\vec{r}_{12}) = i \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q} \cdot \vec{r}_{12}} \left[dI_1^j D'_{jk}(q_0 = 0, \vec{q}) dI_2^k \right]. \quad (12)$$

The $-ig_{\mu\nu}/(q^2 + i\epsilon)$ term in the propagator gives rise to the usual (doubly differential) potential between the current elements $V_0(\vec{r}_{12}) = -\frac{1}{4\pi r_{12}}(d\vec{I}_1 \cdot d\vec{I}_2)$. The calculation follows precisely the same path as the calculation of the Coulomb potential between pointlike charges. However, we are concerned with the novel term,

$$V_\kappa = 2i \frac{\kappa}{1 + \kappa^2} \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q} \cdot \vec{r}_{12}} \frac{1}{\vec{q}^2 - i\epsilon} [(d\vec{I}_1 \times d\vec{I}_2) \cdot \vec{q}]. \quad (13)$$

Splitting $d\vec{I}_1 \times d\vec{I}_2$ into its components parallel and normal to \vec{r}_{12} , only $(d\vec{I}_1 \times d\vec{I}_2)_\parallel = [(d\vec{I}_1 \times d\vec{I}_2) \cdot \hat{r}_{12}] \hat{r}_{12}$ will contribute. This is evident from symmetry considerations alone; the triple product $(d\vec{I}_1 \times d\vec{I}_2) \cdot \vec{r}_{12}$ is the only pseudoscalar that can be constructed from

$d\vec{I}_1 \times d\vec{I}_2$. Moreover, it is easy to see explicitly that for every direction \hat{q} , there is another one \hat{q}' , such that $\vec{q} \cdot \vec{r}_{12} = \vec{q}' \cdot \vec{r}_{12}$, yet $(d\vec{I}_1 \times d\vec{I}_2)_\perp \cdot \hat{q} = -(d\vec{I}_1 \times d\vec{I}_2)_\perp \cdot \hat{q}'$, where $(d\vec{I}_1 \times d\vec{I}_2)_\perp = d\vec{I}_1 \times d\vec{I}_2 - (d\vec{I}_1 \times d\vec{I}_2)_\parallel$; and this leads to complete cancellation in the $(d\vec{I}_1 \times d\vec{I}_2)_\perp$ part of the integral.

As in the evaluation of the nonrelativistic Coulomb potential, we evaluate the integral in spherical coordinates and use $\hat{r}_{12} \cdot \hat{q} = \cos \theta$. This gives

$$V_\kappa(\vec{r}_{12}) = \frac{i}{2\pi^2} \frac{\kappa}{1 + \kappa^2} (d\vec{I}_1 \times d\vec{I}_2) \cdot \hat{r}_{12} \int_0^\infty dQ \frac{Q^2}{Q^2 - i\epsilon} \int_{-1}^1 d(\cos \theta) \cos \theta e^{iQr_{12} \cos \theta}, \quad (14)$$

where Q denotes $|\vec{q}|$. The $i\epsilon$ prescription in the denominator is unneeded, and performing the angular integration, we have

$$V_\kappa(\vec{r}_{12}) = \frac{1}{2\pi^2} \frac{\kappa}{1 + \kappa^2} (d\vec{I}_1 \times d\vec{I}_2) \cdot \hat{r}_{12} \left[\frac{2}{r_{12}^2} \int_0^\infty dQ \frac{\sin(Qr_{12}) - Qr_{12} \cos(Qr_{12})}{Q^2} \right] \quad (15)$$

$$= \frac{1}{\pi^2} \frac{\kappa}{1 + \kappa^2} \frac{1}{r_{12}} (d\vec{I}_1 \times d\vec{I}_2) \cdot \hat{r}_{12}. \quad (16)$$

Evidently, by endowing right- and left-circularly polarized photons with different couplings to charged matter, we have also introduced a new potential between perpendicular currents. The structure of V_κ clearly indicates that it is capable of generating physical forces. For example, two current-carrying wires running in perpendicular directions but not intersecting will feel an attractive or repulsive force, depending on the sign of $(\vec{I}_1 \times \vec{I}_2) \cdot \hat{r}$, where \hat{r} is directed between the wires' points of closest approach.

In fact, for long wires of length L , oriented so that current I_1 flows in the x -direction and I_2 flows in the y -direction, with a closest approach separation $\vec{r} = (0, 0, z)$ between their midpoints, the force is

$$F = -\frac{d}{dz} \left[\frac{1}{\pi^2} \frac{\kappa}{1 + \kappa^2} \int_{-L/2}^{L/2} dx_1 \int_{-L/2}^{L/2} dy_2 \frac{I_1 I_2 z}{x_1^2 + y_2^2 + z^2} \right] \quad (17)$$

$$\approx -\frac{d}{dz} \left[\frac{1}{\pi} \kappa I_1 I_2 z \int_{-L/2}^{L/2} dx_1 \frac{1}{\sqrt{x_1^2 + z^2}} \right] \quad (18)$$

$$\approx -\frac{2}{\pi} \kappa I_1 I_2 \log \frac{L}{z}, \quad (19)$$

for $0 < z \ll L$.

The detailed structure of the potential V_κ at $\mathcal{O}(\kappa^2)$ depends on the particular relation (7). However, the behavior at leading order in κ is free of any ambiguities associated with the choice of normalization. If the right- and left-handed couplings were normalized differently, then in addition to the $\mathcal{O}(\kappa)$ contribution to V_κ , there would also be changes at $\mathcal{O}(\kappa^2)$ to the usual potential between parallel current elements.

5 Experimental Constraints

It should be possible to test for the presence of κ in several different ways. Since κ is known to be small, it is reasonable to work to leading order in κ in examining these tests. One test is quite obvious. Since the two helicities of light couple differently to moving charges, there would be a systematic ellipticity in the radiation from what would be expected to be a linearly polarized source. Such a source would emit radiation with a true ellipticity of $|\kappa|$. This would be detected as an apparent ellipticity of $2|\kappa|$, because the polarization that is produced more weakly is also more weakly coupled to the detector. (If a field redefinition is used to move the Lorentz violation into the Maxwell propagation Lagrangian, then $2|\kappa|$ becomes the true ellipticity. The observable effect is the same in either case.)

Synchrotron radiation is frequently used for calibrating x-ray polarimeters. The Compton polarimeter described in [16] is not sensitive to the relative phases of the perpendicular polarization components it measures, so it interprets elliptically polarized radiation as having a fictitious linear polarization. The plane of this fictitious linear polarization can be identified to within 4×10^{-3} radians accuracy (assuming conventional electrodynamics), and the measurements made with this device are in agreement with the standard predictions at this level. This sets a limit of $|\kappa| \lesssim 2 \times 10^{-3}$.

More accurate polarimetric measurements have been made in experiments looking at photon birefringence (ref. [17] measured ellipticities at the 10^{-7} radians level), but these are typically not sensitive to κ . There are systematic effects (such as stress-induced birefringence in the optical windows that open onto sample cells) with the same experimental signatures as κ that must be subtracted away; such subtraction obviously eliminates any sensitivity to κ . However, the kinds of apparatus used to make these birefringence measurements might be adapted to look for a nonzero κ .

There are also potential magnetostatic tests. By running two perpendicular current-carrying wires close together and measuring the forces they exert on one-another, it would be possible to test for the presence of the force (19). Searches for this kind of novel force could perhaps be combined with searches for other manifestations of Lorentz violation in electromagneticstatics, which can involve preferred direction effects and mixing between electric and magnetic sources and fields [18].

For comparison, the best laboratory bound on the Lorentz-violating coefficient $\tilde{\kappa}_{\text{tr}}$ —which is another isotropic boost invariance violation parameter that may be introduced into the photon sector—are at the 5×10^{-15} level [19]. In the presence of $\tilde{\kappa}_{\text{tr}}$, the speed of light becomes $1 - \tilde{\kappa}_{\text{tr}}$, and this affects the rate of synchrotron emission by charged particles. The 5×10^{-15} bound derives from an analysis of synchrotron losses in energy calibration data from the Large Electron-Positron Collider (LEP). The difference in sensitivities to κ and $\tilde{\kappa}_{\text{tr}}$ has a straightforward explanation. Any experimental constraint on κ requires a measurement of a P-odd observable. Consequently, many types of experiments are insensitive to κ . For example, the energy loss by a lepton beam during a full revolution

around the LEP ring is unaffected by κ .

The current-current interactions deriving from κ also have minimal effects on the mutual interactions of magnetic moments. The field of a circulating current is well described by a magnetic dipole at distances large compared to the size of the current loop. At these distances, the currents on opposite sides of the loop make almost equal and opposite contributions to any force exerted on the dipole due by the new current-current interaction. Conversely, a dipolar field will exert minimal novel forces on other currents in the vicinity; for idealized pointlike dipoles, the κ -dependent forces cancel completely. This makes it difficult to constrain κ with precision atomic experiments, which frequently measure the interactions between dipoles, and this insensitivity is a further consequence of the P-odd character of the κ interaction.

Direct measurements of the effects of $D'_{\mu\nu}$ in scattering experiments may be similarly difficult. It is impossible to test for a purely timelike κ^μ in the center of mass frame of a two-body collision. The momenta \vec{p}_1 and \vec{p}_2 of the incoming particles must be equal and opposite; when the corresponding currents are contracted with the ϵ -tensor in the effective photon propagator, they produce a vanishing result.

In summary, we have introduced a new interaction that couples right- and left-circularly polarized photons to moving charges differently. Although the helicity of a single photon is invariant under rotations and Lorentz boosts, the new interaction still violates Lorentz invariance, because of the way it interacts with electrostatic effects. The interaction is also nonlocal, for a similar reason. However, the propagator for the modified theory (which we have determined exactly) is transverse and describes only two propagating photon modes per wave vector. The propagator gives rise to such novel effects as forces between perpendicular currents. The best bounds on κ , which parameterizes the strength of the helicity-dependent coupling, come from precision polarimetry of synchrotron radiation and are at the 2×10^{-3} level.

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